05

I. (1)

$$\dfrac{\dfrac{\dfrac{[\Phi\vee\Psi]_4 \quad \dfrac{[\Phi]_1 \quad \Phi\to(\chi\vee r)}{\chi\vee r} \quad \dfrac{[\Psi]_1 \quad \Psi\to(\chi\vee r)}{\chi\vee r}}{\chi\vee r}\,1 \qquad \dfrac{\dfrac{[\neg\chi\wedge\neg r]_3}{\neg\chi} \quad [\chi]_2}{\bot} \quad \dfrac{\dfrac{[\neg\chi\wedge\neg r]_3}{\neg r} \quad [r]_2}{\bot}}{\bot}\,2}{\neg(\neg\chi\wedge\neg r)}\,3}{(\Phi\vee\Psi)\to\neg(\neg\chi\wedge\neg r)}\,4$$

(2)

$$\dfrac{\dfrac{\dfrac{[\Phi\wedge(\chi\vee r)]_2}{\chi\vee r} \qquad \dfrac{\dfrac{\dfrac{\dfrac{[\Phi\wedge(\chi\vee r)]_2}{\Phi} \quad \Phi\to\neg\Psi}{\neg\Psi} \quad \neg\Psi\to\neg\chi}{\neg\chi} \quad [\chi]_1}{\bot} \qquad \dfrac{\dfrac{\dfrac{[\Phi\wedge(\chi\vee r)]_2}{\Phi} \quad \Phi\to\neg\Psi}{\neg\Psi} \quad \dfrac{[r]_1 \quad r\to\neg\neg\Psi}{\neg\neg\Psi}}{\bot}}{\bot}\,1.}{\Psi}}{(\Phi\wedge(\chi\vee r))\to\Psi}\,2$$

(3)

$$
\AxiomC{$[\Phi(x)]_1$}
\UnaryInfC{$\exists x\,\Phi(x)$}
\AxiomC{$\exists x\,\Phi(x)\to\Omega$}
\BinaryInfC{Ω}
\UnaryInfC{$\forall x\,\neg\Phi(x)\vee\Omega$}
\AxiomC{$\neg(\forall x\,\neg\Phi(x)\vee\Omega)$}
\BinaryInfC{$\bot$}
\RightLabel{1}
\UnaryInfC{$\neg\Phi(x)$}
\UnaryInfC{$\forall x\,\neg\Phi(x)$}
\UnaryInfC{$\forall x\,\neg\Phi(x)\vee\Omega$}
\AxiomC{$[\neg(\forall x\,\neg\Phi(x)\vee\Omega)]_2$}
\BinaryInfC{$\bot$}
\RightLabel{2}
\UnaryInfC{$\neg\neg(\forall x\,\neg\Phi(x)\vee\Omega)$}
\RightLabel{NK}
\UnaryInfC{$\forall x\,\neg\Phi(x)\vee\Omega$}
\DisplayProof
$$

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(4)

$$
\AxiomC{$\exists x(\exists y\,\Psi(x,y)\to\forall z\,\chi(x,z))$}
\AxiomC{$[\forall x\,\Psi(x,y)]_2$}
\UnaryInfC{$\Psi(u,y)$}
\UnaryInfC{$\Psi(u,w)$}
\AxiomC{$[\forall x\,\Psi(x,y)]_2$}
\UnaryInfC{$\Psi(u,y)$}
\UnaryInfC{$\exists y\,\Psi(u,y)$}
\AxiomC{$[\exists y\,\Psi(u,y)\to\forall z\,\chi(u,z)]_1$}
\BinaryInfC{$\forall z\,\chi(u,z)$}
\UnaryInfC{$\chi(u,w)$}
\BinaryInfC{$\Psi(u,w)\wedge\chi(u,w)$}
\UnaryInfC{$\exists w(\Psi(u,w)\wedge\chi(u,w))$}
\UnaryInfC{$\exists u\exists w(\Psi(u,w)\wedge\chi(u,w))$}
\RightLabel{1}
\BinaryInfC{$\exists u\exists w(\Psi(u,w)\wedge\chi(u,w))$}
\RightLabel{2}
\UnaryInfC{$\forall x\,\Psi(x,y)\to\exists u\exists w(\Psi(u,w)\wedge\chi(u,w))$}
\UnaryInfC{$\forall y(\forall x\,\Psi(x,y)\to\exists u\exists w(\Psi(u,w)\wedge\chi(u,w)))$}
\DisplayProof
$$

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II. Ax. 3-2

$$\forall z \forall x (z + Sx = S(z+x))$$

$$\forall x (z + Sx = S(z+x))$$

$$z + Sx = S(z+x) \qquad [z + Sx = z + Sy]^{1}$$

$$S(z+x) = z + Sy$$

Ax. 3-2

$$\forall z \forall y (z + Sy = S(z+y))$$

$$\forall y (z + Sy = S(z+y))$$

$$z + Sy = S(z+y)$$

$$S(z+x) = S(z+y)$$

Ax. 2

$$S(z+x) = S(z+y) \to z+x = z+y$$

$$z + x = z + y$$

$$z+x = z+y \to x = y$$

$$x = y$$

$$Sx = Sx$$

$$Sx = Sy$$

$$z + Sx = z + Sy \to Sx = Sy \quad 1$$

$$\forall z (z + Sx = z + Sy \to Sx = Sy)$$

//

Ⅲ 部分証明図 Π_1.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(3)}{\forall u(u\in{\downarrow}x \to u\in S)}}{u\in{\downarrow}x \to u\in S} \quad [u\in{\downarrow}x]_2}{u\in S} \quad \dfrac{\dfrac{\dfrac{(3)}{\forall v(v\in{\downarrow}x \to v\in S)}}{v\in{\downarrow}x \to v\in S} \quad [v\in{\downarrow}x]_1}{v\in S}}{u\in S \land v\in S} \quad \dfrac{\dfrac{(1)}{\forall v((u\in S \land v\in S)\to sup(\{u,v\})\in S)}}{(u\in S\land v\in S)\to sup(\{u,v\})\in S}}{sup(\{u,v\})\in S}$$

部分証明図 Π_2

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(4)}{\forall u(u\in{\downarrow}x \to u\leq x)}}{u\in{\downarrow}x \to u\leqq x} \quad [u\in{\downarrow}x]_2}{u\leqq x} \quad \dfrac{\dfrac{\dfrac{(4)}{\forall v(v\in{\downarrow}x \to v\leq x)}}{v\in{\downarrow}x \to v\leqq x} \quad [v\in{\downarrow}x]_1}{v\leqq x}}{u\leqq x \land v\leqq x} \quad \dfrac{\dfrac{(2)}{\forall x((u\leq x \land v\leq x)\to sup(\{u,v\})\leq x)}}{(u\leqq x\land v\leqq x)\to sup(\{u,v\})\leqq x}}{sup(\{u,v\})\leqq x}$$

全体証明図.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(5)}{(sup(\{u,v\})\in S \land sup(\{u,v\})\leqq x)\to sup\{u,v\}\in{\downarrow}x} \quad \dfrac{\overset{\Pi_1}{sup(\{u,v\})\in S} \quad \overset{\Pi_2}{sup(\{u,v\})\leqq x}}{sup(\{u,v\})\in S\land sup(\{u,v\})\leq x}}{sup\{u,v\}\in{\downarrow}x}\,1}{v\in{\downarrow}x\to sup\{u,v\}\in{\downarrow}x}\,2}{u\in{\downarrow}x\to(v\in{\downarrow}x\to sup(\{u,v\})\in{\downarrow}x)}}{\dfrac{\forall v(u\in{\downarrow}x\to(v\in{\downarrow}x\to sup(\{u,v\})\in{\downarrow}x))}{\forall u\forall v(u\in{\downarrow}x\to(v\in{\downarrow}x\to sup(\{u,v\})\in{\downarrow}x))}}$$